Introduction to the Transportation Network Design

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Contents

• Motivation
• Network terminology
• Introduction
• Objective
• Traffic assignment
  – All-or-Nothing assignment,
  – Incremental assignment,
  – User-Equilibrium (UE) assignment,
  – System Optimum (SO) assignment,
  – Stochastic User-Equilibrium (SUE) assignment,
• Solution to the NDP
Contents

• Motivation
• Network terminology
• Introduction
• Objective
• Traffic assignment
  – All-or-Nothing assignment,
  – Incremental assignment,
  – User-Equilibrium (UE) assignment,
  – System Optimum (SO) assignment,
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Motivation

Social and economical development

Importance of networks
(water supply, energy supply, communication, transportation)

Transportation network design
Motivation

- Increasing socio-economical needs
- Varied activities
- Mobility demand
- Private car ownership and usage

Transportation network design
Motivation
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• Objective
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  – All-or-Nothing assignment,
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Network Terminology

- A transport network may be formally represented as a set of *links* and a set of *nodes*.
- A link connects two nodes and a node connects two or more links.
- Links may be either *directed* or *undirected*. 
Network Terminology

- A movement in a transportation network corresponds to a flow with a distinct Origin (O) and Destination (D).
- O-Ds may correspond to specific buildings like a house or and office, or to zones (districts).
- O-Ds are represented by a kind of node, referred to as a centroid.
- Each centroid is connected to one or more nodes by a kind of link referred to as a connector.
Network Terminology

• Links may have various characteristics.

• In the context of transportation network analysis, the following are some of the characteristics of interest:
  
  – Link length (in meters or perhaps in average vehicles)
  – Link cost (generally travel time) and
  – Link capacity (maximum flow)
Contents

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• Network terminology
• Introduction
• Objective
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  – All-or-Nothing assignment,
  – Incremental assignment,
  – User-Equilibrium (UE) assignment,
  – System Optimum (SO) assignment,
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Introduction

- Transportation **Network Design Problem** (NDP) concerns the configuration of a network to achieve specified objectives.
Transportation network design
Transportation network design
Transportation network design
NDP

Lane addition

Transportation network design
NDP

Building reversible lanes

Transportation network design
NDP

Congestion pricing

Transportation network design
NDP

Widening traffic lanes

Transportation network design
NDP

Road resurfacing

Transportation network design
NDP

Signal controlled intersections

Transportation network design
NDP

Intelligent transportation systems

Transportation network design
Introduction

• There are two forms of the problem
  – The continuous network design problem
  – The discrete network design problem
Introduction

• The continuous network design problem takes the network topology as given and is concerned with the parameterization of the network. For example:

  – The determination of road width (number of lanes);

  – The calculation of traffic signal timings;

  – The setting of user charges (public transport fares, road tolls, etc)
The discrete network design problem is concerned with the topology of the network. For example:

- A road closure scheme;

- The provision of a new public transport service (represented as a new set of links); and

- The construction of a new road rail link, perhaps a bridge, a tunnel or a bypass.
Introduction

User benefits

- Construction of an extra lane (increases the capacity of the road)
- Allocation of more capacity to one signal-controlled link by increasing green time per cycle
- Imposition or increasing user charges (can bring about benefits to society as a whole through greater economic efficiency)

Cost of network alteration

- Construction of an extra lane (brings cost)
- Reduces the capacity of some other link(s) through reduced green time per cycle
- Imposition or increasing user charges (may result in extra costs for certain users)

TRADE-OFF
Contents

• Motivation
• Network terminology
• Introduction
• Objective
• Traffic assignment
  – All-or-Nothing assignment,
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Objective

- Traditional network design has been concerned with the **minimization of system cost** (equal to the sum of link flows times link costs).

\[
SC = \sum_{i \in I} v_i c_i \left( v_i, s_i \right)
\]

- \( c_i \): the cost (travel time) on link \( i \)
- \( v_i \): the flow on link \( i \) (**TRAFFIC ASSIGNMENT**) 
- \( s_i \): the value of the design parameter for link \( i \)

Transportation network design
Contents

• Motivation
• Network terminology
• Introduction
• Objective
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  – Incremental assignment,
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  – System Optimum (SO) assignment,
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Traffic Assignment

• The process of allocating given set of trip interchanges to the specified transportation system is usually referred to as traffic assignment.

• The fundamental aim of the traffic assignment process is to reproduce on the transportation system, the pattern of vehicular movements which would be observed when the travel demand represented by the trip matrix, or matrices, to be assigned is satisfied.
Traffic Assignment

Transportation network design
Traffic Assignment

The major aims of traffic assignment procedures are:

- To estimate the volume of traffic on the links of the network,
- To obtain aggregate network measures, e.g. total vehicular flows, total distance covered by the vehicle, total system travel time.
- To estimate zone-to-zone travel costs(times) for a given level of demand.
- To obtain reasonable link flows and to identify heavily congested links.
- To estimate the routes used between each origin to destination(O-D) pair.
- To obtain turning movements for the design of future junctions.
Traffic Assignment

- The types of traffic assignment models:
  - All-or-Nothing assignment,
  - Incremental assignment,
  - User-Equilibrium (UE) assignment,
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Contents

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• Network terminology
• Introduction
• Objective
• Traffic assignment
  – All-or-Nothing assignment,
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All-or-Nothing Assignment

- The trips from any origin zone to any destination zone are loaded onto a single, minimum cost, path between them.

\[ q_{1-4} = 2000 \text{ veh/h} \]

<table>
<thead>
<tr>
<th>Path no</th>
<th>Route</th>
<th>Free flow travel time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-3-4</td>
<td>6 mins</td>
</tr>
<tr>
<td>2</td>
<td>1-2-4</td>
<td>8 mins</td>
</tr>
<tr>
<td>3</td>
<td>1-2-3-4</td>
<td>10 mins</td>
</tr>
<tr>
<td>4</td>
<td>1-3-2-4</td>
<td>12 mins</td>
</tr>
</tbody>
</table>

\[ f_{Path_{-1}} = 2000 \text{ veh/h} \]
All-or-Nothing Assignment

\[ t_{Route_1} = 79 \text{ seconds} \]
\[ t_{Route_2} = 69 \text{ seconds} \]
\[ t_{Route_3} = 119 \text{ seconds} \]

\[ D_{2-4} = 1000 \text{ veh/h} \]

Transportation network design
All-or-Nothing Assignment

\[ t_{\text{Route}_1} = 79 \text{ seconds} \]
\[ v_{\text{Route}_1} = 0 \]
\[ t_{\text{Route}_2} = 136 \text{ seconds} \]
\[ v_{\text{Route}_2} = 1000 \text{ veh/h} \]
\[ t_{\text{Route}_3} = 149 \text{ seconds} \]
\[ v_{\text{Route}_3} = 0 \]

Transportation network design
All-or-Nothing Assignment

• This model is unrealistic as:

  – Only one path between every Origin-Destination (O-D) pair is utilized even if there is another path with the same or nearly same travel cost.

  – Traffic on links is assigned without consideration of whether or not there is adequate capacity or heavy congestion.
All-or-Nothing Assignment

• This model may be used:
  – For uncongested networks where there are a few alternative routes and they have a large difference in travel cost (travel time)
  – To identify the desired path which the drivers would like to travel in the absence of congestion
Incremental Assignment

• This is a process in which fractions of traffic volumes are assigned in steps.

• In each step, a fixed proportion of total demand is assigned, based on all-or-nothing assignment.

• After each step, link travel times are recalculated based on link volumes.
Incremental Assignment

$t_{Route_1} = 79$ seconds  
$t_{Route_2} = 69$ seconds  
$t_{Route_3} = 119$ seconds

$D_{2-4} = 1000$ veh/h

Transportation network design
Incremental Assignment

$t_{Route_1}=98$ seconds
$v_{Route_1}=500$ veh/h

$t_{Route_2}=85$ seconds
$v_{Route_2}=500$ veh/h

$t_{Route_3}=126$ seconds
$v_{Route_3}=0$

Transportation network design
Incremental Assignment

\[ t_{\text{Route}_1} = 92 \text{ seconds} \]
\[ v_{\text{Route}_1} = 400 \text{ veh/h} \]
\[ t_{\text{Route}_2} = 93 \text{ seconds} \]
\[ v_{\text{Route}_2} = 600 \text{ veh/h} \]
\[ t_{\text{Route}_3} = 130 \text{ seconds} \]
\[ v_{\text{Route}_3} = 0 \]
Contents

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- Network terminology
- Introduction
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Wardrop’s Principles

- **John Glen Wardrop** (1920 - 1989) was an English transport analyst who developed **Wardrop's first and second principles** of equilibrium.

- The concepts are related to the idea of **Nash equilibrium** in **game theory** developed separately. However, in transportation networks, there are many players, making the analysis complex.
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Wardrop's Principles

- **Wardrop's first principle** of route choice became accepted as a sound and simple behavioral principle to describe the spreading of trips over alternate routes due to congested conditions.

- Wardrop's first principle states: *The journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route.*
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\[ D_{12} = 50 \]

\[ v_1 = 21, \quad t_1 = 21 \]

\[ v_2 = 29, \quad t_2 = 21 \]

\[ v_3 = 0, \quad t_3^0 = 24 \]

\[ T_{TT} = 21 \times 21 + 29 \times 21 + 0 \times 24 = 1050 \]
Wardrop’s Principles

• Each user **non-cooperatively** seeks to minimize his cost of transportation. The traffic flows that satisfy this principle are usually referred to as "user equilibrium" (UE) flows, since each user chooses the route that is the best.

• Specifically, a user-optimized equilibrium is reached when no user may lower his transportation cost through unilateral action.

\[
D_{12} = 50
\]

\[
v_1 = 21, t_1 = 21
\]

\[
v_2 = 29, t_2 = 21
\]

\[
v_3 = 0, t_3^0 = 24
\]

\[
TTT = 21 \times 21 + 29 \times 21 + 0 \times 24 = 1050
\]
Wardrop’s Principles

- **Wardrop's second principle** states: *At equilibrium the average journey time is minimum.*
- This implies that each user behaves **cooperatively** in choosing his own route to ensure the most efficient use of the whole system.
- Traffic flows satisfying Wardrop's second principle are generally deemed "system optimal" (SO).

\[ D_{12} = 50 \]

\[ v_1 = 18, \quad t_1 = 23 \]
\[ v_2 = 32, \quad t_2 = 19 \]
\[ v_3 = 0, \quad t_3 = 24 \]

\[ TTT = 18 \times 23 + 32 \times 19 + 0 \times 24 = 1022 \]
User-Equilibrium (UE)

- The user equilibrium assignment is based on Wardrop's first principle, which states that no driver can unilaterally reduce his/her travel costs by shifting to another route.

- For each O-D pair, at user equilibrium, the travel time on all used paths is equal, and (also) less than or equal to the travel time that would be experienced by a single vehicle on any unused path.

- If it is assumed that drivers have perfect knowledge about travel costs on a network and choose the best route according to Wardrop's first principle, this behavioral assumption leads to deterministic user equilibrium.
User-Equilibrium (UE)

- This problem is equivalent to the following nonlinear mathematical optimization program,

\[ \min z = \sum_{a \in A} \int_{0}^{v_a} t_a(x) \, dx \]  

(1)

s.t.

\[ \sum_{k \in K} f_{k}^{rs} = D_{rs} \quad \forall r \in R, \ s \in S, \ k \in K_{rs} \]  

(2)

\[ v_a = \sum_{rs} \sum_{k \in K_{rs}} f_{k}^{rs} \delta_{a,k} \quad \forall r \in R, \ s \in S, \ a \in A, k \in K_{rs} \]  

(3)

\[ f_{k}^{rs} \geq 0 \quad \forall r \in R, \ s \in S, \ k \in K_{rs} \]  

(4)

\( k \) is the path, \( v_a \) equilibrium flows on link \( a \), \( t_a \) travel time on link \( a \), \( f_{k}^{rs} \) flow on path \( k \) connecting O-D pair \( r-s \), \( D_{rs} \) trip rate between \( r \) and \( s \).
User-Equilibrium (UE)

\[ t_{\text{Route}_1} = 79 \text{ seconds} \]
\[ t_{\text{Route}_2} = 69 \text{ seconds} \]
\[ t_{\text{Route}_3} = 119 \text{ seconds} \]

\[ D_{2-4} = 1000 \text{ veh/h} \]
User-Equilibrium (UE)

\[ t_{\text{Route}_1} = 92 \text{ seconds} \]
\[ v_{\text{Route}_1} = 410 \text{ veh/h} \]

\[ t_{\text{Route}_2} = 92 \text{ seconds} \]
\[ v_{\text{Route}_2} = 590 \text{ veh/h} \]

\[ t_{\text{Route}_3} = 129 \text{ seconds} \]
\[ v_{\text{Route}_3} = 0 \]

Transportation network design
System Optimum (SO)

- The system optimum assignment is based on Wardrop's second principle, which states that drivers cooperate with one another in order to minimize total system travel time.

- This assignment can be thought of as a model in which congestion is minimized when drivers are told which routes to use.

- Obviously, this is not a behaviorally realistic model, but it can be useful to transport planners and engineers, trying to manage the traffic to minimize travel costs and therefore achieve an optimum social equilibrium.
System Optimum (SO)

- This problem is equivalent to the following nonlinear mathematical optimization program,

\[
\min z = \sum_a v_a \cdot t_a (v_a)
\]  

(5)

s.t.

\[
\sum_{k \in K} f_{rs}^{rs} = D_{rs} \quad \forall r \in R, \ s \in S, \ k \in K_{rs}
\]  

(6)

\[
\nu_a = \sum_{rs} \sum_{k \in K_{rs}} f_{rs}^{rs} \delta_{rs}^{rs} \quad \forall r \in R, \ s \in S, \ a \in A, k \in K_{rs}
\]  

(7)

\[
f_{rs}^{rs} \geq 0 \quad \forall r \in R, \ s \in S, \ k \in K_{rs}
\]  

(8)

\(k\) is the path, \(\nu_a\) equilibrium flows on link \(a\), \(t_a\) travel time on link \(a\), \(f_{rs}^{rs}\) flow on path \(k\) connecting O-D pair \(r-s\), \(D_{rs}\) trip rate between \(r\) and \(s\).
System Optimum (SO)

\[ t_{\text{Route}_1} = 79 \text{ seconds} \]
\[ t_{\text{Route}_2} = 69 \text{ seconds} \]
\[ t_{\text{Route}_3} = 119 \text{ seconds} \]

\[ D_{2-4} = 1000 \text{ veh/h} \]
System Optimum (SO)

$t_{Route_1} = 86$ seconds
$v_{Route_1} = 354 \text{ veh/h}$

$t_{Route_2} = 85$ seconds
$v_{Route_2} = 646 \text{ veh/h}$

$t_{Route_3} = 126$ seconds
$v_{Route_3} = 0$

Assignment

<table>
<thead>
<tr>
<th>Assignment</th>
<th>TTT</th>
</tr>
</thead>
<tbody>
<tr>
<td>AoN</td>
<td>136000</td>
</tr>
<tr>
<td>UE</td>
<td>92000</td>
</tr>
<tr>
<td>SO</td>
<td>85354</td>
</tr>
</tbody>
</table>
Example - I

- Demonstration of the common assignment methods.
- This network has two nodes having two path as links with constant travel time.
- Total flows from 1 to 2, \( D_{12} = 12 \) vehicles

**Figure 1:** Two Link Problem with constant travel time function
Example - I

- **All-or-Nothing**
  - The travel time functions for both the links is given by:
    
    \[ t_1 = 10 \]
    \[ t_2 = 15 \]

  - Total flows from 1 to 2:
    
    \[ D_{12} = 12 \]

  - Since the shortest path is Link 1, all flows are assigned to it making \( v_1^* = 12 \) and \( v_2^* = 0 \).
Example - I

- **User Equilibrium**
  - Substituting the travel time in Equations 1-4 yield to:

  \[
  \min z = \sum_{a \in A} \int_{t_a(x)}^{v_a} t_a(x) dx = \int_{0}^{v_1} 10 \cdot dv_1 + \int_{0}^{v_2} 15 \cdot dv_2 \\
  = 10 \cdot v_1 + 15 \cdot v_2 \\
  \]

  subject to: \( v_1 + v_2 = 12 \)

  - Substituting \( v_2 = 12 - v_1 \), in the above formulation will yield the unconstrained formulation as below:

  \[
  \min z = 10 \cdot v_1 + 15(12 - v_1) \\
  \]

  - Differentiate the above equation with respect to \( v_1 \) and equate to zero, and solving for \( v_1 \) and then \( v_2 \) leads to the solution \( v_1^* = 12 \) and \( v_2^* = 0 \)
Example - I

• **System Optimum**
  
  – Substituting the travel time in Equations 5-8, we get the following:

  \[
  \min z = v_1 \times 10 + v_2 \times 15 \\
  = 10 \cdot v_1 + 15 \cdot v_2
  \]

  – Substituting \( v_2 = 12 - v_1 \), the above formulations take the following form:

  \[
  \min z = 10 \cdot v_1 + 15(12 - v_1)
  \]

  – Differentiate the above equation with respect to \( v_1 \) and equate to zero, and solving for \( v_1 \) and then \( v_2 \) leads to the solution \( v_1^* = 12, v_2^* = 0 \) and \( z(v^*) = 120 \).
Comparison of results

- After solving each of the formulations the results are given in the following table.

<table>
<thead>
<tr>
<th>Type</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$v_1$</th>
<th>$v_2$</th>
<th>$Z(x^*)$</th>
<th>TTT</th>
</tr>
</thead>
<tbody>
<tr>
<td>AON</td>
<td>10</td>
<td>15</td>
<td>12</td>
<td>0</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>UE</td>
<td>10</td>
<td>15</td>
<td>12</td>
<td>0</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>SO</td>
<td>10</td>
<td>15</td>
<td>12</td>
<td>0</td>
<td>120</td>
<td>120</td>
</tr>
</tbody>
</table>
Example - II

• This network has two nodes having two path as links with travel times as functions of the link flow.

• Total flows from 1 to 2, $D_{12} = 12$ vehicles

Figure 1: Two Link Problem with constant travel time function
Example - II

- **All-or-Nothing**
  - Assume that $v_1, v_2 = 0$ which makes $t_1 = 10$ and $t_2 = 15$.

- Since the shortest path is Link 1, all flows are assigned to it making $v_1^* = 12$ and $v_2^* = 0$. 
Example - II

• **User Equilibrium**

  – Substituting the travel time in Equations 1-4 yield to:

  \[
  \min z = \sum_{a \in A} \int_0^{v_a} t_a(x) \, dx = \int_0^{v_1} (10 + 3v_1) \, dv_1 + \int_0^{v_2} (15 + 2v_2) \, dv_2
  \]

  \[
  = 10v_1 + \frac{3v_1^2}{2} + 15 \cdot v_2 + \frac{2v_2^2}{2}
  \]

  subject to: \( v_1 + v_2 = 12 \)

  – Substituting \( v_2 = 12 - v_1 \), in the above formulation will yield the unconstrained formulation as below:

  \[
  \min z = 10v_1 + \frac{3v_1^2}{2} + 15(12 - v_1) + \frac{2(12 - v_1)^2}{2}
  \]

  – Differentiate the above equation with respect to \( v_1 \) and equate to zero, and solving for \( v_1 \) and then \( v_2 \) leads to the solution \( v_1^* = 5.8 \) and \( v_2^* = 6.2 \)
Example - II

- **System Optimum**
  
  - Substituting the travel time in Equations 5-8, we get the following:

  \[
  \min z = v_1 \times (10 + 3v_1) + v_2 \times (15 + 2v_2)
  \]

  \[
  = 10 \cdot v_1 + 3v_1^2 + 15 \cdot v_2 + 2v_2^2
  \]

  - Substituting \( v_2 = 12 - v_1 \), the above formulations take the following form:

  \[
  \min z = 10 \cdot v_1 + 3v_1^2 + 15 \cdot (12 - v_1) + 2(12 - v_1)^2
  \]

  - Differentiate the above equation with respect to \( v_1 \) and equate to zero, and solving for \( v_1 \) and then \( v_2 \) leads to the solution \( v_1^* = 5.3, \ v_2^* = 6.7 \) and \( z(v^*) = 327.55 \)
Comparison of results

- After solving each of the formulations the results are given in the following table.

<table>
<thead>
<tr>
<th>Type</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$v_1$</th>
<th>$v_2$</th>
<th>$Z(x^*)$</th>
<th>TTT</th>
</tr>
</thead>
<tbody>
<tr>
<td>AON</td>
<td>10</td>
<td>15</td>
<td>12</td>
<td>0</td>
<td>467.4</td>
<td>552</td>
</tr>
<tr>
<td>UE</td>
<td>27.4</td>
<td>27.4</td>
<td>5.8</td>
<td>6.2</td>
<td>239.0</td>
<td>329</td>
</tr>
<tr>
<td>SO</td>
<td>30.1</td>
<td>25.6</td>
<td>5.3</td>
<td>6.7</td>
<td>327.5</td>
<td>328</td>
</tr>
</tbody>
</table>
Contents

- Motivation
- Network terminology
- Introduction
- Objective
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  - All-or-Nothing assignment,
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A variant on Wardrop’s first principle (UE) is the stochastic user equilibrium (SUE), wherein no driver can unilaterally change routes to improve his/her perceived travel times.

In the SUE, the assumption that travelers have perfect information on the road network is relaxed.

Travelers experience perception error. The SUE behavioral assumption brings some advantages over DUE in that it represents behavior more realistically.
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• Introduction
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  – All-or-Nothing assignment,
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Solution to the NDP

• In a road network to be optimized, there is an interaction between the design parameters and the routes chosen by individual road users.

• The problem falls within the framework of a leader-follower (or Stackelberg) game, where the designer (i.e. transport planner) is the leader and the user is the follower (Fisk, 1984).

• When drivers follow Wardrop's (1952) first principle, (i.e UE), the problem is called the “equilibrium network design problem (ENDP), which is normally non-convex.
Solution to the NDP

• In the literature, this leader-follower game has widely been formulated as a combined bi-level problem.

• The bi-level programming (BLP) problem is a special case of multilevel programming problems with a two level structure.

• The problem can be expressed as follows: the transport planner, wishes to optimize the control variables in the upper level, and the users response to these controls in UE manner in the lower level.
Solution to the NDP

• In the last decades, several solution methods have been developed to solve bi-level ENDP.

• These methods can be classified as deterministic and stochastic in nature.

• The deterministic methods can be classified as the linearization, sensitivity analysis and gradient based local search approaches that require substantial gradient information to find a solution.

• Stochastic methods, in particular genetic algorithms, differential evolution, harmony search, ant colony optimization, etc., have widely been employed to the solution of the ENDP.
Modified differential evolution algorithm for the continuous network design problem

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Assoc. Prof. Dr. Huseyin CEYLAN

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Pamukkale University
Denizli / Turkey

Transportation network design
Content

✓ Objectives and problem definition
✓ Background
✓ Differential evolution algorithm
✓ Numerical application
✓ Results
✓ Findings and future studies
Objectives

This study aims

• To develop a new solution method, which is capable to find near optimal solutions of the CNDP within less computational time

• To improve the base DE algorithm in order to facilitate the solution process.
Problem definition

CNDP is

• To determine the set of link capacity expansions
• To find corresponding equilibrium link flows
and ...

Considering both objectives,

“The measure of performance index for the network should be optimal”
Problem definition....

\[
\begin{align*}
\min_{x,y} \quad & Z(x, y) = \sum_{a \in A} (t_a(x_a, y_a)x_a + \rho g_a(y_a)) \quad \text{Upper level} \\
\text{s.t.} \quad & 0 \leq y_a \leq u_a , \quad \forall a \in A \\
\end{align*}
\]

\[
\begin{align*}
\min_{x} \quad & z = \sum_{a \in A} \int_{0}^{x_a} t_a(w, y_a)dw \quad \text{Lower level} \\
\text{s.t.} \quad & \sum_{k \in K} f_{rs}^{k} = D_{rs} \quad \forall r \in R, \ s \in S, \ k \in K_{rs} \\
\end{align*}
\]

\[
\begin{align*}
x_a = \sum_{rs} \sum_{k \in K_{rs}} f_{rs}^{k} \delta_{a,k} \quad \forall r \in R, \ s \in S, \ a \in A, k \in K_{rs} \\
f_{rs}^{k} \geq 0 \quad \forall r \in R, \ s \in S, \ k \in K_{rs}
\end{align*}
\]

Transportation network design
# Background

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<th>Decision</th>
<th>Lower level</th>
<th>Method</th>
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## Transportation network design
Differential Evolution Algorithm

- DE is a simple and powerful algorithm which is introduced by Storn and Price (1995).
- Three control parameters are used to control optimization process.
  - *Number of populations* (NP)
  - *Mutation factor* (F)
  - *Crossover rate* (CR)
The steps of the DE algorithm

• *Generation of the initial population*
• *Mutation*
• *Crossover*
• *Selection*
Improvement mechanisms

First improvement on the standard DE is to take different mutation strategies into account.

\[
m_{i}^{j,t} = \begin{cases} 
    y_{i}^{1,t} + F(y_{i}^{2,t} - y_{i}^{3,t}), & \text{if rand (0,1) } < \text{ MSCR} \\
    y_{i}^{1,t} + F(y_{i}^{\text{best},t-1} - y_{i}^{2,t}), & \text{otherwise}
\end{cases}
\]

**Aim of this improvement**

“To take the effect of best solution vector determined at previous generation into account”
Differential Evolution Algorithm...

The second improvement mechanism is a kind of embedding a local search mechanism to the DE algorithm.

Aim of this improvement

“To push the best solution existed in the population towards to the global or near global optimum one step closer”
Numerical application

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Transportation network design
Numerical application...

The travel time function is:

\[ t_a(x_a, y_a) = \alpha_a + \beta_a \left( \frac{x_a}{\theta_a + y_a} \right)^4 \]

The upper level objective function is defined as:

\[
\min_{x,y} Z(x,y) = \sum_{a \in A} \left( t_a(x_a, y_a)x_a + d_a y_a \right)
\]

s.t. \[ 0 \leq y_a \leq u_a \quad , \quad \forall a \in A \]
## Results

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### Scenario 1

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### Values

- **Z**: 190.33
- **NUE**: 396

**Transportation network design**
Results...

Convergence graph for Scenario 1

- DE
- MODE

Transportation network design
Results...

Convergence graph for Scenario 2

Transportation network design
Findings and future studies

• The improved version of the base DE achieved better solutions in all scenarios in terms of objective function value and number of UE assignments in comparison with the DE, SA and GA.

• The developed mutation strategy and local search operator have facilitated the rate of convergence of the base DE algorithm.
Findings and future studies...

“It is necessary to test the developed algorithm for large-scale road network applications”
Thank you for your attention